CHAPTER 8

2.1 Vertex Features

[illegible]

[illegible]

3 OUTCOMES ON NETWORKS

… $\beta < 1$, … $\mathbf{I} - \beta W$ … $(\alpha, \beta, \eta, \gamma)$ …:

Proposition 1 *If* $\beta < 1$, $\eta\beta + \gamma = 0$, $W_{ij} = (N-1)^{-1}$ *if* $i = j$ *and* $W_{ii} = 0$, $(\alpha, \beta, \eta, \gamma)$ *is not point-identified.*

… (1993) … P… 1 … B… D… F… (2009). … K… P… (2006) … (1993). … (1), …

$$y_i = \alpha + \beta \mathbb{E}(y_j \; w) + \eta x_i + \gamma \mathbb{E}(x_j \; w) + \; _i, \quad \mathbb{E}(\quad =$$

j

x_i. A … M (1993) … (p. 535). … P… 1 … x, … $\mathbb{V}(\mathbb{E}(x_j\ w)) = 0$, …

A … (2) … I … $\mathbb{V}(\ \mathbf{x}) = \sigma^2\mathbf{I}$, …

$$\mathbb{V}(\mathbf{y}\ \mathbf{x}) = \sigma^2(\mathbf{I} - \beta W)^{-2}.$$

… β … $W_{ij} = (N-1)^{-1}$ … $i = j$ … $W_{ii} = 0$. I … M (2001) … (F…, 1966; B… P…, 1986; H…, N…, … 1987). B…, I … N … [12]

Proposition 2 *If* $\beta < 1$, $W_{ij} = (N-1)^{-1}$ *if* $i = j$, $W_{ii} = 0$, *and* $\mathbb{V}(\ \mathbf{x}) = \sigma^2\mathbf{I}$ *then* $(\alpha, \beta, \eta, \gamma)$ *is point-identified.*

I … N … 3. … : I … i' … β … j. I … k … i' … k … j' … i' … (O… $N = 2$ … -1. … $N > 2$.[13]) A … (…), P… 2 …

[12] C…

[13] … $N = 3$, … P… 3 … -0.45. F… -0.50.

Proposition 3 *If* $\beta < 1$, $W_{ij} = (N-1)^{-1}$ *if* $i = j$, $W_{ii} = 0$, *and* $\mathbb{V}(\ \mathbf{x}) = \sigma^2\mathbf{I}$ *then*

$$\frac{\mathbb{C}(y_i, y_j\ \mathbf{x})}{\mathbb{V}(y_i\ \mathbf{x})} > \frac{4-3N}{4N^2-11N+8}.$$

[illegible] P [illegible] 3 [illegible] $l = 1, \ldots, L$, [illegible] α_l), D[illegible], d'H[illegible] [illegible] F[illegible] (2009) [illegible] $\mathbb{V}(\ \mathbf{x}) = \sigma^2\mathbf{I}$ [illegible] *at least* [illegible] P[illegible] 3.2). [illegible] (2015) [illegible] $\mathbb{V}(\ \mathbf{x}) = \sigma^2\mathbf{I} + \sigma\ (W + W\ )$. [illegible] W [illegible] B[illegible], D[illegible] [illegible] F[illegible] (2009) [illegible]).

[illegible] (2) [illegible] G[illegible], 1996 [illegible]). G[illegible] (2008), [illegible] $l = 1, \ldots, L$ [illegible]

$$\mathbf{y}_{l\ N_l\ 1} = \gamma W_{l\ N_l\ N_l\ l\ N_l\ 1} + \alpha_l \mathbf{1}_{N_l\ 1} + \ _{l\ N_l\ 1},$$

[illegible] N_l [illegible] l, $W_{ij,l} = (N_l-1)^{-1}$ [illegible] $i = j$ [illegible] $W_{ii,l} = 0$, [illegible] α_l [illegible].[14] (A[illegible] G[illegible], 2003.) [illegible] $_{i,l}$, [illegible] $\sum_{j=i}\ _{j,l}/(N_l-1)$, [illegible] α_l. [illegible]

[illegible] (2015) [illegible] (2) [illegible] 5).

A [illegible] (2) [illegible] B[illegible], D[illegible], [illegible] F[illegible] (2009):

Proposition 4 (B[illegible], D[illegible], [illegible] F[illegible], 2009**)** *If* $\eta\beta + \gamma = 0$ *and* $\mathbf{I}$, W, W^2 *are linearly independent,* $(\alpha, \beta, \eta, \gamma)$ *is point-identified.*

I[illegible] $W_{ij} = (N-1)^{-1}$ [illegible] $i = j$ [illegible] $W_{ii} = 0$, $W^2 = (N-1)^{-1}\mathbf{I} +$

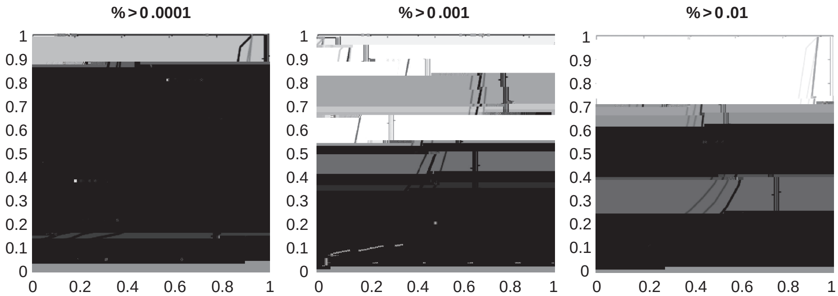
F te 1 De é d C é Ne .

e e . Te a W^2 t a $(W^2$

[illegible]

[illegible]

[illegible]



F e 3 ... De é d C é . *Note:* Te d e e , e d, d d e e e $(\mathbf{I} - \beta W)^{-1}(\mathbf{I} + W)$ e 0.0001, 0.001 d 0.01, e e e β. Te W 100 100 $(N = 100)$, $W_{ii+1} = W_{100,1} = 1$ $i = 1, \ldots, 100$ d e , e e . Te d (e e e) W 1 e e , d e d $\mathbf{I} + W$, e d $\beta = 0$, 2 e e .

e e d, e b . A *approximate* e e é de e e e d e e e (i, j) e W^k e ,

$$(W,\beta,\delta,\gamma)\ \frac{1}{T}\sum_{t}\|\mathbf{y}_t-\alpha-\beta W\mathbf{y}_t-\eta\mathbf{x}_t-\gamma W\mathbf{x}_t\|_2^2+\lambda\sum_{i=j}p_T(W_{ij}), \tag{6}$$

[illegible] (e.g., CAD MCP). [illegible].[28]

[illegible] (2015) [illegible]. [illegible] (2015; [illegible].)

3.2 Nonlinearities and Multiple Equilibria

[illegible] (e.g., [illegible], 2011). [illegible], *non-mutually exclusive*, [illegible] (e.g., $y_i = f\left(\sum^N \right.$ [illegible]

d b t e e a t e $\mathbf{y}(\mathbf{x}), \mathbf{x}$ 0, 1 ^{N}

4 NETWORK FORMATION

4.1 Statistical Models

(*but not necessarily more than one*).

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] $N_i(g)$ [illegible] i [illegible] $(\ _{ij})_{j=i}$ [illegible] W_{ij} [illegible] 0, 1 [illegible] i [illegible] j [illegible]

[illegible]

[illegible]

[illegible]

A PROOFS

A.1 Proof of Proposition 2

[illegible] $\mathbb{V}(\ \mathbf{x}) = \sigma^2\mathbf{I}$, [illegible] $\mathbb{V}(\mathbf{y}\ \mathbf{x}) = \sigma^2(\mathbf{I} - \beta W)^{-2}$. [illegible] $\beta < 1$ [illegible] W [illegible]

$$(\mathbf{I} - \beta W)^{-1} = \mathbf{I} + \beta W + \beta^2 W^2 + \dots$$

[illegible] $W^k, k = 1, 2, \dots$ [illegible] $(W^k)_{ii} = a_{k-1}$ [illegible] $(W^k)_{ij} = a_k, i \neq j$, [illegible]

$$a_0 = 0, \qquad a_{-1} = 1 \text{ [illegible] } a_k = (a_{k-2} + a_{k-1}(N-2))/(N-1).$$

[illegible] $S = \sum_{k=1} \beta$

$\mathbb{V}(\mathbf{y}\ \mathbf{x}) = \sigma^2(\mathbf{I} + S)^2,$

O e e d, e t d e e $p(b; \overline{\kappa}, N)$ e d

$$\overline{\kappa} > \frac{-(N-2)}{(N-2)^2 +}$$

$$\psi(\beta) = \frac{\mathbb{C}(y_{i,1}, y_{j,1}\ \mathbf{x}_1)}{\mathbb{C}(y_{i,2}, y_{j,2}\ \mathbf{x}_1)} = \frac{8 + 8\beta + 2\beta^2}{4 + 7\beta + 2\beta^2 - \beta^3}.$$

I [illegible] $\psi(\beta) < 0$ [illegible] $\beta < 1$ [illegible] $\psi(\beta) > \psi(1) = 1.5$.

[illegible] $\overline{\psi}$ [illegible] β [illegible] $q(b; \overline{\psi}) = 0$ [illegible] $q(b; \overline{\psi})$ [illegible] $-\overline{\psi}\beta^3 + 2(\overline{\psi} - 1)\beta^2 + (7\overline{\psi} - 8)\beta + 4(\overline{\psi} - 2)$. [illegible] 1 [illegible] (1987, [illegible] 225 [illegible] 229). [illegible] f [illegible] g [illegible] C [illegible] $g(x) < f(x)$ [illegible] C, [illegible] f [illegible] $f + g$ [illegible] C, [illegible] $f(x) = a_1 x$ ([illegible] $f(x) = 0$ [illegible] $x = 0$) [illegible] $g(x) = a_0 + a_2 x^2 + \cdots + a_K x^K$ [illegible]

( ) T ee e [illegible] $(g = ij,$

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