

## INFERENCE OF SIGNS OF INTERACTION EFFECTS IN SIMULTANEOUS GAMES WITH INCOMPLETE INFORMATION

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T. ... W. ... T. ... (DGP). A ... DGP. W. ... U ... S.

... (S ... (2009)), ...  
... (B ..., H ..., K ...,  
... N ... (2010)).  
E ...

... W ... S ... 3 ...

. T  
BNE. T BNE  
I T

(CDF)  $F_{i|x}(\cdot|x)$ . Then, for  $i \in N$ ,  $x \in X$ ,  $u_i(x)$  is the utility function of player  $i$  at  $x$ .  $D_j$  is the distribution of  $S_j$  given  $x$ .  $I_i$  is the indicator function of the set  $\{x \in X : u_i(x) + v_i(x) \geq \mathbb{E}[S_j(x_j)|x] - v_i(x)\}$ .  $M_i(x)$  is the measure of player  $i$  at  $x$ .  $T$  is the set of all  $(u_i)_{i=1}^N$ .  $W$  is the set of all  $(F_{i|x})_{i=1}^N$ .

ASSUMPTION 1:  $x \in X, F_{i|x}(\cdot|x) = \prod_{i \leq N} F_{i|x}(\cdot|x)$  for all  $x \in X$ .

A.  $X = X$ .  $I_i$  is the indicator function of the set  $\{x \in X : u_i(x) + v_i(x) \geq \mathbb{E}[S_j(x_j)|x] - v_i(x)\}$ .  $T$  is the set of all  $(u_i)_{i=1}^N$ .  $B$  is the set of all  $(S_i)_{i=1}^N$ .  $S_i: X \rightarrow \{0, 1\}$ .  $L$  is the set of all  $(S_i)_{i=1}^N$ .

$$S_i(x_j) = \begin{cases} 1 & \text{if } u_i(x) + v_i(x) \geq \mathbb{E}[S_j(x_j)|x] - v_i(x), \\ 0 & \text{otherwise.} \end{cases}$$

U. A.  $1, \mathbb{E}[S_j(x_j)|x = x_j] = \mathbb{E}[S_j(x_j)|x = x] \equiv p_j(x)$ ,  $B$  is the set of all  $(S_i)_{i=1}^N$ . (BNE)  $p(x) \equiv [p_1(x) \dots p_N(x)]$  for all  $x \in X$ .

$$(1) \quad p_i(x) = F_{i|x=x} u_i(x) + v_i(x) p_j(x) \quad i=1 \dots N$$

$p_i(x)$  is the probability of player  $i$  choosing  $1$  given  $x$ .  $F_{i|x}$  is the cumulative distribution function of  $S_i$  given  $x$ .  $X$  is the set of all  $x \in X$ .  $L$  is the set of all  $(S_i)_{i=1}^N$ .  $BNE$  is the set of all  $(u_i)_{i=1}^N$ .  $p$  is the vector of probabilities  $p_j(x)$ .  $(1)$  is the system of equations (1).  $B$  is the set of all  $(S_i)_{i=1}^N$ .  $A$  is the set of all  $(u_i)_{i=1}^N$ .  $I_i$  is the indicator function of the set  $\{x \in X : u_i(x) + v_i(x) \geq \mathbb{E}[S_j(x_j)|x] - v_i(x)\}$ .  $T$  is the set of all  $(u_i)_{i=1}^N$ .  $N$  is the set of all  $(i, j)$ .

(see also [3, Section 3.1] and [1, Section 3.1]). The first case is the case of the  $\mathbb{R}^n$ -valued function  $f_1(x) = \sum_{j \neq i} (D_j)$ , where  $D_j$  is the  $j$ -th component of the vector  $D$ . The second case is the case of the  $\mathbb{R}^n$ -valued function  $f_1(x) = \sum_{j \neq i} (D_j)$ , where  $D_j$  is the  $j$ -th component of the vector  $D$ .



PROOF: Under Assumption 1,  $D_i$  is a convex set, and  $\sum_{j \neq i} D_j$  is a convex set. Let  $x \in \text{BNE}$ ,  $p^i \in \mathcal{L}_x^+$ . For any  $f \in \mathcal{F}$ ,  $f(\cdot) \cdot S$  is a linear function, and  $\text{BNE}$  implies  $f(p^i(x)) \cdot S \geq f(p^j(x)) \cdot S$  for all  $p^j \in \mathcal{L}_x^+$ . Let  $p^i(x) = p_i^i(x)$ ,  $p^j(x) = \sum_{j \neq i} p_j^j(x)$ , and  $\tilde{p}_i^*(x) = p_i^i(x) \sum_{j \neq i} p_j^j(x)$ . Then  $\tilde{p}_i^*(x) = p_i^i(x) \cdot p^j(x)$ . Let  $f(\cdot) \cdot S$  be a linear function, and  $\mathcal{L}_x^+$  is a convex set. Let  $p^i \neq p_i^i \cdot A$ , then  $p^i(x) \cdot S < p_i^i(x) \cdot S$ .  $\square$

$$(3) \quad p_i^i(x) \equiv \tilde{p}_i^*(x) - p_i^i(x) \cdot p^j(x)$$

$$= \int_{p^i \in \mathcal{L}_x^+} p_i^i(x) \cdot p^j(x) d_x - \int_{p^i \in \mathcal{L}_x^+} p_i^i(x) d_x \int_{p^i \in \mathcal{L}_x^+} p^j(x) d_x$$

Since  $p_i^i(x) > 0$ , then  $\int_{p^i \in \mathcal{L}_x^+} p_i^i(x) \cdot p^j(x) d_x = \int_{p^i \in \mathcal{L}_x^+} h_i(p_i^i(x)) \cdot p^j(x) d_x = \int_{p^i \in \mathcal{L}_x^+} h_i(p_i^i(x)) \cdot p^j(x) d_x = \int_{p^i \in \mathcal{L}_x^+} (F_{i|x}^{-1}(p_i^i(x)) - u_i(x)) / p_i^i(x) \cdot p^j(x) d_x$ .  $\square$

$$\tilde{p}_i^*(x) - p_i^i(x) \cdot p^j(x)$$

$$= \int_0^1 h_i(z) z d_{\tilde{p}_i^*(x)} - \int_0^1 z d_{\tilde{p}_i^*(x)} \int_0^1 h_i(z) d_{\tilde{p}_i^*(x)}$$

Let  $Z \equiv p_i^i(x) \cdot \tilde{p}_i^*(x)$ . Then  $Z \in \mathcal{L}_x^+$ . Then  $\int_0^1 h_i(z) z d_{\tilde{p}_i^*(x)} = \int_0^1 h_i(z) z d_Z$ .  $\square$

$$\int_0^1 h_i(z) z d_Z = \mathbb{E} Z - \mathbb{E}(Z) \cdot h_i(\mathbb{E}(Z))$$

$$= \mathbb{E} (Z - \mathbb{E}(Z)) \cdot h_i(\mathbb{E}(Z))$$

$$+ \mathbb{E} (Z - \mathbb{E}(Z)) \cdot (h_i(\mathbb{E}(Z)) - \mathbb{E}(h_i(Z)))$$

$$= \mathbb{E} (Z - \mathbb{E}(Z)) \cdot h_i(\mathbb{E}(Z))$$

By Assumption 1,  $h_i$  is strictly increasing on  $[0, 1]$ . Let  $x$ ,  $z_1 > z_2 \Rightarrow h_i(z_1) > h_i(z_2)$ . Condition C implies  $(z - \mathbb{E}(Z))(h_i(z) - h_i(\mathbb{E}(Z))) > 0$  for all  $z \neq \mathbb{E}(Z)$ . Then  $\int_0^1 h_i(z) z d_{\tilde{p}_i^*(x)} - \int_0^1 z d_{\tilde{p}_i^*(x)} \int_0^1 h_i(z) d_{\tilde{p}_i^*(x)} > 0$ .  $\square$

<sup>7</sup>Then  $\int_0^1 h_i(z) z d_{\tilde{p}_i^*(x)} = \int_0^1 h_i(z) z d_Z$ .  $\square$





T.  $\dots (i, x)$   $\dots$   
 $\dots (i, x)$ . I  $\dots$   
 $\dots (i, x)$ . W  $\dots$

$f_i(x)$   
 C  $\dots$   
 $x \in X$   $\dots$  i. T.  $\dots (i, \cdot)$   $\dots$   
 $\dots$  BNE  $\dots$  DGP  $\dots$   
 $F_X$ . T.  $\dots$   $i(x) > (<) 0$   $\dots$   $\tilde{p}_i^*(x) - p_i^*(x)$   $\dots$   $i^*(x) > (<)$   
 0. F  $\dots$   $i(x) > (<) 0$   $\dots$  DGP  $\dots$   
 $x$ ,  $\dots$

$\tilde{X}_0$  ...  $\tilde{X}_i$  ...  $F_{j|x}$  ...  $F_{j|x_i}$  ...  $x_i = (\tilde{X}_0 \tilde{X}_i)$  ...  $F_{j|x=x} = F_{j|x=x_i}$  ...

ASSUMPTION 2: ...  $f \times (\dots X_i)$

$u_i(x) = u_i(x_i), \quad v_i(x) = v_i(x_i), \quad F_{j|x=x} = F_{j|x=x_i} f \dots x.$

T...  $(v_i(x))$  ...  $x)$  ...  $u_j(x')$  ...  $F_{j|x=x'}$  ...  $v_j(x')$  ...  $x' = v_j(x_i)$  ...

1) ...  $x' = v_j(x_i)$  ...  $x' = v_j(x_i)$  ...  $v_j(x_i)$  ...  $x' = v_j(x_i)$  ...

M. C. (S. 5). W. P. L. g. D. i. g. D.

$$\begin{aligned}
 i(x_i) &\equiv \mathbb{E} D_{i,g} \quad D_{j,g} \quad X_g \in i(x_i) \\
 &\quad j \neq i \\
 &= \mathbb{E}[D_{i,g} | X_g \in i(x)] \mathbb{E} \quad D_{j,g} \quad X_g \in i(x_i) \\
 &\quad j \neq i
 \end{aligned}$$

PROPOSITION 2: 1 2 ( ) x, ( i(x)) = ( i(x\_i)) f i f \* x\_i i' ( ) \* x\_i i' f f i(x\_i) ≠ 0.

PROOF: C (i x) \* x\_i T. (1) A 2 h\_i(z) = h\_i(p\_i(z)) z ∈ i(x\_i) p\_i ∈ L\_x^+, h\_i(·) ≡ (F\_{i|x}^{-1}(·) - u\_i(x))/ i(x). T. h\_i j ≠ i. L \* x\_i i' BNE.

$$\begin{aligned}
 (5) \quad i(x_i) &= \mathbb{E} D_i \quad D_j \quad p \quad X \in i(x_i) \quad d * x_i \\
 &\quad p \in \mathcal{L}_{x_i}^* \quad j \neq i \\
 &= \mathbb{E}[D_i | p \quad X \in i(x_i)] \quad d * x_i \\
 &\quad p \in \mathcal{L}_{x_i}^* \\
 &= \mathbb{E} \quad D_j \quad p \quad X \in i(x_i) \quad d * x_i \\
 &\quad p \in \mathcal{L}_{x_i}^* \quad j \neq i \\
 &= \sum_{p \in \mathcal{L}_x^*} p_i \quad p_j \quad d * x_i - \sum_{p \in \mathcal{L}_x^*} p_i \quad d * x_i \quad \sum_{p \in \mathcal{L}_x^*} p_j \quad d * x_i \\
 &\quad j \neq i \quad j \neq i
 \end{aligned}$$

p ∈ [0 1]^N

$f_i(x) > 0$  ( $< 0$ ),  $x \in X_i(x_i)$ . Here  $P$  is a  $1 \times 1$  matrix,  $P_{ii} > 0$  ( $< 0$ ),  $f_i(x) > 0$  ( $< 0$ ),  $x_i^* \in X_i(x_i)$ .  
 (6)  $f_i(x) > 0$  ( $< 0$ ),  $x_i^* \in X_i(x_i)$ .

Theorem 2. For  $f_i(x) \neq 0$ ,  $x_i^* \in X_i(x_i)$ ,  $f_i(x) > 0$  ( $< 0$ ),  $x_i^* \in X_i(x_i)$ .  
 (7)  $f_i(x) > 0$  ( $< 0$ ),  $x_i^* \in X_i(x_i)$ .

Theorem 3. For  $f_i(x) > 0$  ( $< 0$ ),  $x_i^* \in X_i(x_i)$ .

**EXAMPLE 2.** Let  $A_i(x) = C_i(x) - 2x_i - 2x_j$ ,  $C_i(x) = 1 - 2x_i - 2x_j$ ,  $X_i = [0, 1]$ ,  $X_j = [0, 1]$ ,  $X_0 = [0, 1]$ ,  $X_1 = [0, 1]$ ,  $X_2 = [0, 1]$ . Let  $S_i(x) = 1 - 2x_i - 2x_j$ ,  $f_i(x) = 1 - 2x_i - 2x_j$ . Let  $X_1 \equiv (X_0, X_1)$ ,  $X_2 \equiv (X_0, X_2)$ . Let

... i... i... x  
... \* i?  
... x  
BNE... X.

#### 4. TESTING MULTIPLE BNE AND INFERRING INTERACTION SIGNS

W...  
D... (2007, .58). H... BNE... T...  
) (.) W...  
N...  
H...  
A... (

I ... S ... 3 ...  $f_i(x) \neq 0$  ... i ... -  
 x. I ... S ... M ... x. W ... (7)  
 ( )  $f_i(x, D_{-i})$  ... DGP  
 A ... (1) ... u ... F<sub>x</sub>.<sup>11</sup> I ... BNE ... -  
 N ... (7).  
 W ... S ... 4.1 ...  
 F ... N = 2, ... BNE ... x ... I ... -  
 11

for  $i = 1, \dots, N$  and  $x \in \mathbb{R}^d$ :

$$H_i^0: \beta_i(x) = 0$$

$$H_i^1: \beta_i(x) \neq 0$$

Given a set  $P \subset \{1, \dots, N\}$ , we define the  $P$ -FWE as follows:  $f_{P, \alpha} = \inf_{\beta \in \mathcal{B}_P} \max_{i \in P} \int_{\mathbb{R}^d} \beta_i(x) dx$  (FWE), where  $\mathcal{B}_P = \{\beta \in \mathcal{B} : \beta_i(x) \geq 0 \text{ for } i \in P\}$ . Then,

$$\text{FWE}_P = \mathbb{P}_P\{\beta \in \mathcal{B}_P : H_i^0: \beta_i(x) = 0 \text{ for } i \in I_0(P)\}$$

where  $\mathbb{P}_P$  is the probability measure induced by the DGP,  $I_0(P) \subset \{1, \dots, N\}$  is the set of indices  $i$  such that  $\beta_i(x) = 0$  for all  $x \in \mathbb{R}^d$ . P. A.

$$\text{FWE}_P \leq \int_{\mathbb{R}^d} \max_{i \in P} \beta_i(x) dx \quad \text{as } G \rightarrow +\infty \quad \text{FWE}_P \leq \int_{\mathbb{R}^d} \max_{i \in P} \beta_i(x) dx \quad \text{P.}$$

W.  $\int_{\mathbb{R}^d} \max_{i \in P} \beta_i(x) dx$   $\int_{\mathbb{R}^d} \max_{i \in P} \beta_i(x) dx$

S.  $\int_{\mathbb{R}^d} \max_{i \in P} \beta_i(x) dx$   $\int_{\mathbb{R}^d} \max_{i \in P} \beta_i(x) dx$   $X = x$  W.

$\int_{\mathbb{R}^d} \max_{i \in P} \beta_i(x) dx$   $\int_{\mathbb{R}^d} \max_{i \in P} \beta_i(x) dx$   $X$

( $\dots$ ). S.  $\int_{\mathbb{R}^d} \max_{i \in P} \beta_i(x) dx$   $\int_{\mathbb{R}^d} \max_{i \in P} \beta_i(x) dx$

$\int_{\mathbb{R}^d} \max_{i \in P} \beta_i(x) dx$   $\int_{\mathbb{R}^d} \max_{i \in P} \beta_i(x) dx$   $N$



$$\hat{\mu}_G \xrightarrow{d} \mathcal{N}(\mathbf{0}_{\tilde{N}}, G^{1/2}(\hat{\mu}_G(\{x\}) - \mu(\{x\})) \xrightarrow{d} \mathcal{N}(\mathbf{0}_{\tilde{N}}, \Sigma(\{x\})) \quad G \rightarrow \infty,$$

$$\mathbf{D} = \mathbf{T}_G(\{x\}) \quad \mathbf{N} = \mathbf{I}_{\tilde{N}}$$

$$T_{G,i}(\{x\}) \equiv \frac{\hat{\rho}_{ij}(\{x\})}{\hat{\rho}_0(\{x\})} - \frac{\hat{\rho}_i(\{x\})\hat{\rho}_j(\{x\})}{(\hat{\rho}_0(\{x\}))^2}$$

**B**  $G^{1/2} \mathbf{T}_G(\{x\}) - \Delta(x) \xrightarrow{d} \mathbf{N} \mathbf{0}_N \mathbf{V}(\{x\}) \Sigma(\{x\}) \mathbf{V}(\{x\})'$

$$G^{1/2} \mathbf{T}_G(\{x\}) - \Delta(x) \xrightarrow{d} \mathbf{N} \mathbf{0}_N \mathbf{V}(\{x\}) \Sigma(\{x\}) \mathbf{V}(\{x\})'$$

$$G \rightarrow \infty$$

$$\Delta(x) \equiv ( \mu_i(x) )_{i=1}^N, \mathbf{T} = \mathbf{J} \quad \mathbf{V}(\{x\}) = \mathbf{N}^{-1} \tilde{\mathbf{N}}^{-1} \mathbf{I}_i$$

$$V_i(\{x\}) = \mu_{(m)}(\{x\}) V_{i(m)}(\{x\})$$

$$m = \tilde{N} \quad \mu(\{x\}) = V_i(\{x\}),$$

$$j \neq k \neq i):$$

$$\frac{\mu_{(m)}(\{x\})}{\mu_0(\{x\})}: \quad V_{i(m)}(\{x\})$$

$$\sum_{j \neq i} \left( -\frac{\mu_{ij}(\{x\})}{\mu_0(\{x\})^2} + \frac{2\mu_i(\{x\})\mu_j(\{x\})}{\mu_0(\{x\})^3} \right)$$

$$\mu_i(\{x\}): \quad -\sum_{j \neq i} \frac{\mu_j(\{x\})}{\mu_0(\{x\})^2}$$

$$\mu_j(\{x\}): \quad -\frac{\mu_i(\{x\})}{\mu_0(\{x\})^2}$$

$$\mu_{ij}(\{x\}) \quad \mu_{ji}(\{x\}): \quad \frac{1}{\mu_0(\{x\})}$$

$$\mu_{jk}(\{x\}): \quad 0$$

**W**  $\Sigma(\{x\}) \mathbf{V}(\{x\}) \mathbf{F} \quad \mu_0(\{x\}) \quad \mu_i(\{x\})$

**W**  $\mathbf{B} \quad \mathbf{H} \quad \mathbf{B}$

$\rho_i$   $\rho_i$   $\hat{\rho}_{G,i}$   $\mathbf{T} \quad \mathbf{B}$

$\hat{\rho}_{G,i} \leq \rho_i / N$   $\mathbf{T} \quad \mathbf{H}$   $\mathbf{F}$

$\mathbf{B}$   $\mathbf{H}$   $\rho_i$

$\hat{\rho}_{G(1)} \leq \hat{\rho}_{G(2)} \leq \dots \leq \hat{\rho}_{G(N)}$   $\mathbf{L} \quad \mathbf{H}_{jk}^0$



<sup>15</sup>  $W$   $\hat{c}_k$   $S$  -

$$\begin{aligned}
& \text{where } \mathbf{S}_i = \mathbf{S}_i(\mathbf{x}_i) \text{ and } \mathbf{V}_i = \mathbf{V}_i(\mathbf{x}_i), \\
& \hat{\mathbf{V}}_i(\mathbf{x}_i) \hat{\boldsymbol{\Sigma}}_i(\mathbf{x}_i) \hat{\mathbf{V}}_i(\mathbf{x}_i)' / G^{-1/2} \\
& \quad \times \mathbf{T}_{G_i}(\mathbf{x}_i) - \mathbf{x}_i \xrightarrow{d} \mathcal{N}(0, \mathbf{1}) \quad \text{as } G \rightarrow \infty \\
& \text{where } \hat{\mathbf{V}}_i(\mathbf{x}_i) = \mathbf{V}_i(\mathbf{x}_i) \text{ and } \hat{\boldsymbol{\Sigma}}_i(\mathbf{x}_i) = \boldsymbol{\Sigma}_i(\mathbf{x}_i), \\
& \mathbf{T}_{G_i}(\mathbf{x}_i) = \mathbf{S}_i(\mathbf{x}_i) \mathbf{V}_i(\mathbf{x}_i) \mathbf{V}_i(\mathbf{x}_i)' \mathbf{S}_i(\mathbf{x}_i) \quad \mathbf{S}_i \text{ as in 4.1.}
\end{aligned}$$

F ( G),

$$D_{ig} = 1 - u_i - W_g \sum_{j \neq i} p_j^1 - (1 - W_g) \sum_{j \neq i} p_j^2 - u_{ig} \geq 0$$

g ≤ G, W\_g, B, N, F ( G), S = 1000  
 F S 4.2  
 W, S 4.2: ( ) (A 3.2 4.2  
 T\_G, ( ) ( )  
 R W (2005)). F  
 ( ) ( ) ( )  
 B). W B = 1000 2000. I T, I, H\_0  
 i = 1) S = 1000 RP1, RP2,  
 RP3, RP

ON EFFECTS

	i = 3
	[0.000, 1.000]
	[0.000, 1.000]
	[0.000,1.000]
	[0.000, 1.000]
	[0.000, 1.000]
	[0.000, 1.000]

Test:  $T = \frac{1}{n} \sum_{i=1}^n \ln \frac{f_i}{g_i}$   
 where  $H_0: f = g$  vs  $H_1: f \neq g$ .

Test:  $T = \frac{1}{n} \sum_{i=1}^n \ln \frac{f_i}{g_i}$   
 where  $H_0: f = g$  vs  $H_1: f > g$ .  
 Test:  $T = \frac{1}{n} \sum_{i=1}^n \ln \frac{f_i}{g_i}$   
 where  $H_0: f = g$  vs  $H_1: f < g$ .

TABLE IV  
FINITE SAMPLE PERFORMANCE: TEST OF SIGNS OF INTERACTION EFFECTS

	G = 5000			G = 10,000		
	= 0.8	= 0.9	= 1.0	= 0.8	= 0.9	= 1.0
$X_1 = -1$	[0 000 0 469]	[0 001 0 628]	[0 000 0 854]	[0 000 0 717]	[0 000 0 890]	[0 000 0 986]
$X_2 = -1/2$	[0 003 0 359]	[0 000 0 520]	[0 000 0 714]	[0 000 0 577]	[0 000 0 790]	[0 000 0 925]
$X_3 = -1$	[0 000 0 483]	[0 000 0 643]	[0 000 0 834]	[0 000 0 702]	[0 000 0 888]	[0 000 0 986]
$X_1 = 2$	[0 323 0 004]	[0 459 0 000]	[0 667 0 000]	[0 484 0 000]	[0 736 0 000]	[0 910 0 000]
$X_2 = 3/2$	[0 400 0 000]	[0 617 0 000]	[0 817 0 000]	[0 665 0 000]	[0 867 0 000]	[0 979 0 000]
$X_3 = 3$	[0 300 0 004]	[0 496 0 000]	[0 735 0 000]	[0 545 0 000]	[0 764 0 000]	[0 930 0 000]

N = 1000, S = 1000. T = 1000.  $q_+ = [q_+, q_-]$ ;  $q_+$  = test of  $H_0$  vs  $H_+$ ;  $q_-$  = test of  $H_0$  vs  $H_-$ .

1. A  $x_1$  strategic, effects a strat-dependent  
 (0 2523 0 5288 0 7098),  $x_1^*(p) = 1/4$ ,  $p \in \{(0 3233 0 5603 0 3233),$   
 (0 2998 0 7013 0 2998), (0 2101 0 7262 0 7231)},  
 T. 1. A  
 $x_1$

1, strategic, effects a strat-dependent  
 from player 1, states the restr

O  
 (CD, TV),  
 A  
 (S (2006))  
 W  
 L  
 S (2009)  
 H ( )  
 B  
 S O  
 B ( )  
 T S (2009, 7)  
 S T  
 W (2001, 24)  
 B  
 F 2:30 (G (1988))  
 H T ( )  
 G  
 F  
 D ( )



## INTERACTION EFFECTS IN SIMULTANEOUS GAMES

TABLE VI  
MULTIPLICITY TESTS (X = HOUR OF DAY)

		15:55	16:55	G
A	W	33.32*		26,152
	RW	$T_1^\dagger > T_3^\dagger > T_2^\dagger > 0$		
N	W	3.86		6534
	RW	$T_3 > T$		

TABLE VII  
 :55 MIN VS. NOT :55 MIN (X = HOUR OF DAY, MARKET SIZE)

Market Size (X)	S	Hour of Day			
		N 1	4 5	5 6	9 10
1	W	0.77	4.94	3.22	2.27
	RW	$T_3 > T_2 > 0 > T_1$	$T_2 > T_1 > T_3 > 0$	$T_2 > T_1 > T_3 > 0$	$T_1 > T_3 > T_2 > 0$
	G	2201	2201	2200	2199
2	W	0.73	3.87	1.97	2.48
	RW	$T_2 > T_3 > 0 > T_1$	$T_3 > 0 > T_1 > T_2$	$T_2 > T_1 > T_3 > 0$	$T_2 > T_1 > T_3 > 0$
	G	2157	2220	2159	2153
3	W	4.96	19.06*	26.07*	2.92
	RW	$T_2 > T_3 > T_1 > 0$	$T_3^\dagger > T_2^\dagger > T_1^\dagger > 0$	$T_1^\dagger > T_3^\dagger > T_2^\dagger > 0$	$T_1 > T_3 > 0 > T_2$
	G	2176	2141	2177	2168

T... (\*)... W... 5%. T... (†)...  
 ... 5% FWE ... R... W... (2005) (RW). T<sub>k</sub> ... k.  
 T... 1000.

4 5 6

7. CONCLUSION

I

**F**  $\mathbb{R}^n$  is a  $\mathbb{R}^n$ -valued function on  $\mathbb{R}^n$ . **I**  $\mathbb{R}^n$  is a  $\mathbb{R}^n$ -valued function on  $\mathbb{R}^n$ .



...  $f$  ... , ...  $f$  ... , 371 ... , ...  
... 1 104, ... .  
... , 2010 ... , 2011.