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INFERENCE OF SIGNS OF INTERACTION EFFECTS IN SIMULTANEOUS GAMES WITH INCOMPLETE INFORMATION

BY UREO DE PAULA AND XUN TANG[1]

[illegible] (DGP). A [illegible] DGP. W [illegible] U [illegible] S [illegible]

, (S (2009))

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N (2010)).

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BNE BNE
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ε -
(CDF) $F_{\varepsilon|X}(\cdot|x)$. T i
1 $U_{1i}(X,\varepsilon_i) \equiv u_i(X) + (\sum_{j\neq i} D_j)\delta_i(X) - \varepsilon_i$,
$U_{0i}(X,\varepsilon_i)$ 0. I , $u_i(X)$
1 i. M $\delta_i(X)$ i'
j 1. T $(u_i,\delta_i)_{i=1}^{N}$
$F_{\varepsilon|X}$
. W $F_{\varepsilon|X}$ -
.

ASSUMPTION 1: $x \in \Omega_X$, $F_{\varepsilon|X}(\cdot|x) = \prod_{i\le N} F_{\varepsilon_i|X}(\cdot|x)$
$\mathbb{R}^N$.

A 1 ε_i' $X = x$. I X -
, . T
. A i
B $s_i : \Omega_{X,\varepsilon_i} \to \{0,1\}$. L $S_i(X,\varepsilon_i)$
i,

$$S_i(X,\varepsilon_i) = \begin{cases} 1, & u_i(X) + \delta_i(X) \sum_{j\neq i} \mathbb{E}[S_j(X,\varepsilon_j)|X,\varepsilon_i] - \varepsilon_i \ge 0, \\ 0, & . \end{cases}$$

U A 1, $\mathbb{E}[S_j(X,\varepsilon_j)|X = x,\varepsilon_i] = \mathbb{E}[S_j(X,\varepsilon_j)|X = x] \equiv p_j(x)$,
B N (BNE) (x)
$p(x) \equiv [p_1(x),\ldots,p_N(x)]$
$x \in \Omega_X$,

(1) $$p_i(x) = F_{\varepsilon_i|X=x}\Big(u_i(x) + \delta_i(x) \sum_{j\neq i} p_j(x)\Big) \quad i = 1,\ldots,N,$$

$p_i(x)$ i' 1
x $F_{\varepsilon_i|X}$ ε_i X. L $\mathcal{L}_{x,\theta}$
BNE (p (1)) x
$\theta \equiv \{(u_i,\delta_i)_{i=1,\ldots,N}, F_{\varepsilon|X}\}$. T - BNE
x B '
$F_{\varepsilon_i|X}$ A 1. I BNE,
$F_{\varepsilon|X}$, u_i, δ_i.

T
(., δ_{ij}).
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P T (2012))). T , ,
( ) ,
(., $f_i(x, D_{-i}) = \ldots_{j\neq i}(D_j)$),
(.,

PROOF: U A 1, D_i $\sum_{j\neq i} D_j$ - x BNE p^l $\mathcal{L}_x$.

f f ( ). S BNE - -; , $\mathcal{L}_x^+$, p^l . T $p_i^*(x) = p_i^l(x)$, $y_i^*(x) = \sum_{j\neq i} p_j^l(x)$, $\tilde{y}_i^*(x) = p_i^l(x)\sum_{j\neq i} p_j^l(x)$ i x. H $\tilde{y}_i^*(x) = p_i^*(x) y_i^*(x)$ i.

f ( ). S $\mathcal{L}_{x,\theta}^+$ x. T i $p^l, p^k \in \mathcal{L}_x^+$ $p_i^l \neq p_i^k$. A i, $\delta_i(x)$. B ,

$$(3) \qquad \Delta_i(x) \equiv \tilde{y}_i^*(x) - p_i^*(x) y_i^*(x) = \int_{p^l\in\mathcal{L}_x^+} p_i^l(x) y_i^l(x)\, d\Lambda_x - \int_{p^l\in\mathcal{L}_x^+} p_i^l(x)\, d\Lambda_x \int_{p^l\in\mathcal{L}_x^+} y_i^l(x)\, d\Lambda_x.$$

S $\delta_i(x) > 0$. T (1) h_i $y_i^l(x) = h_i(p_i^l(x)) \equiv (F_{\varepsilon_i|X}^{-1}(p_i^l(x)) - u_i(x))/\delta_i(x)$ p^l $\mathcal{L}_{x,\theta}$.[7] T x , (3)

$$\tilde{y}_i^*(x) - p_i^*(x) y_i^*(x) = \int_0^1 h_i(z) z\, d\tilde{\Lambda}_{i,x}(z) - \int_0^1 z\, d\tilde{\Lambda}_{i,x}(z) \int_0^1 h_i(z)\, d\tilde{\Lambda}_{i,x}(z),$$

$z \equiv p_i^l(x)$ $\tilde{\Lambda}_{i,x}$ $p_i^l(x)$ Λ_x $\mathcal{L}_x$. T (3) Z :

$$\begin{aligned}(Z, h_i(Z)) &= \mathbb{E}\big[\big(Z - \mathbb{E}(Z)\big)\big(h_i(Z) - \mathbb{E}(h_i(Z))\big)\big]\\ &= \mathbb{E}\big[(Z - \mathbb{E}(Z))\big(h_i(Z) - h_i(\mathbb{E}(Z))\big)\big]\\ &\quad + \mathbb{E}\big[(Z - \mathbb{E}(Z))\big(h_i(\mathbb{E}(Z)) - \mathbb{E}(h_i(Z))\big)\big]\\ &= \mathbb{E}\big[(Z - \mathbb{E}(Z))\big(h_i(Z) - h_i(\mathbb{E}(Z))\big)\big].\end{aligned}$$

B h_i [0, 1] x, $z_1 > z_2 \Rightarrow h_i(z_1) > h_i(z_2)$. C , $(z - \mathbb{E}(Z))(h_i(z) - h_i(\mathbb{E}(Z))) > 0$ $z \neq \mathbb{E}(Z)$, , $\tilde{\Lambda}_{i,x}$ $\mathcal{L}_x^+$. H $\tilde{y}_i^*(x) - p_i^*(x) y_i^*(x) > 0$ BNE - x. T $\delta_i(x) < 0$

[7] T h_i θ x . W .

T … , … $(i|x)$ … , … -
… (… , … , …). I … , … , … -
… $(i|x)$. W … .

… f $\delta_i(x)$

C … $x \in \Omega_X$ … i. T … $(\delta_i(\cdot))$ … i … BNE … DGP … -
… F_X. T … , … $\delta_i(x) > (<) 0$ … $\tilde{y}_i^*(x) - p_i^*(x)y_i^*(x) > (<)$
0. F … , … $\delta_i(x) > (<) 0$ … DGP
… x, …

X $\tilde{X}_0$ - - . X - $(\tilde{X}_i)_{i\leq N}$ $\tilde{X}_i$ - i. F , $\tilde{X}_i$ i. T - $(\varepsilon_i)_{i\leq N}$ - ( ) . I ’ ( ) i’ $\tilde{X}_0$ $\tilde{X}_i$, $F_{\varepsilon_i|X} = F_{\varepsilon_i|X_i}$, $X_i = (\tilde{X}_0, \tilde{X}_i)$. F (i, x), $Y_i(x_i) \equiv \{x' : x'_i = x_i\}$ i x. W .

ASSUMPTION 2: i, f X (X_i) $u_i(x) = u_i(x_i)$, $\delta_i(x) = \delta_i(x_i)$, $F_{\varepsilon_i|X=x} = F_{\varepsilon_i|X_i=x_i}$ f x.

T $(\delta_i(x))$ (i BNE x) : ( ) P i BNE $u_j(x'), \delta_j(x'), F_{\varepsilon_j|X=x'}$ $Y_i(x_i)$. ( ) B , i (1) x' $Y_i(x_i)$. ( ) O ’ - i’ x' $Y_i(x_i)$ i, x' $Y_i(x_i)$ A 2. C , ’ , i

M C (S 5). W -
P 2. L g ,
$D_{i,g}$ i g. D

$$\Psi_i(x_i) \equiv \mathbb{E}\ D_{i,g} \quad D_{j,g} \quad X_g \in Y_i(x_i)$$
$$j \neq i$$

$$- \mathbb{E}[D_{i,g} | X_g \in Y_i(x)] \mathbb{E} \quad D_{j,g}\ X_g \in Y_i(x_i)$$
$$j \neq i$$

PROPOSITION 2: 1 2 . ( ) x,
$(\delta_i(x)) =$ $(\Psi_i(x_i))$ *f* i *f* $\Lambda^*_{x_i}$ i'
( ) $\Lambda^*_{x_i}$ i' *f* *f* $\Psi_i(x_i) \neq 0$.

PROOF: C (i, x) $\Lambda^*_{x_i}$. T
(1) A 2 h_i
$y^l_i(z) = h_i(p^l_i(z))$ $z \in Y_i(x_i)$ $p^l \in \mathcal{L}^+_x$, $h_i(\cdot) \equiv (F^{-1}_{\varepsilon_i|x}(\cdot) -$
$u_i(x))/\delta_i(x)$. T h_i i'
BNE $j \neq i$. I $\Lambda^*_{x_i}$ i' ,

(5) $$\Psi_i(x_i) = \int_{p \in \mathcal{L}^*_{x_i}} \mathbb{E}\ D_i \quad D_j \quad p, X \in Y_i(x_i) \quad d\Lambda^*_{x_i}$$
$$j \neq i$$

$$- \int_{p \in \mathcal{L}^*_{x_i}} \mathbb{E}[D_i | p, X \in Y_i(x_i)]\, d\Lambda^*_{x_i}$$

$$\cdot \int_{p \in \mathcal{L}^*_{x_i}} \mathbb{E} \quad D_j\ p, X \in Y_i(x_i) \quad d\Lambda^*_{x_i}$$
$$j \neq i$$

$$= \int_{p \in \mathcal{L}^*_x} p_i \quad p_j \quad d\Lambda^*_{x_i} - \int_{p \in \mathcal{L}^*_x} p_i\, d\Lambda^*_{x_i} \int_{p \in \mathcal{L}^*_x} \quad p_j \quad d\Lambda^*_{x_i},$$
$$j \neq i \qquad\qquad j \neq i$$

$p \in [0, 1]^N$

$\delta_i(x) > 0$ (< 0) $x \in Y_i(x_i)$. H … P 1 … (i, x), $\Psi_i(x_i) > 0$ (< 0) $\delta_i(x) > 0$ (< 0) … $\Lambda^*_{x_i}$ … i’ … . I … (6) … $\Lambda^*_{x_i}$ … i’ … , … $\Psi_i(x_i) = 0$. … . . .

T … $\Lambda^*_{x_i}$ … i’ … A … 2. F … , … $\delta_i(x) \neq 0$ … $(u_j, \delta_j, F_{\varepsilon_j|X})$ … $j \neq i$ … i. T … i … x ($\delta_i(x) = 0$).[8] P ( ) P 2 … . T … (i, x) … .

EXAMPLE 2 N … A i, x: C … 2- -2 … 1 … 2 … X … $(\tilde{X}_0, \tilde{X}_1, \tilde{X}_2)$, … $\tilde{X}_0$ … - … , … $\tilde{X}_1$ … $\tilde{X}_2$ … - … 1 … 2, … . S … $\delta_i(x) \neq 0$ … i, x, … A … 1 … 2 … $X_1 \equiv (\tilde{X}_0, \tilde{X}_1)$ … $X_2 \equiv (\tilde{X}_0, \tilde{X}_2)$. A …

i i x

, Λ^*_x i'

BNE x.

4. TESTING MULTIPLE BNE AND INFERRING INTERACTION SIGNS

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I S 3 $\square_i(x) \neq 0$ i -
x. I S M ,
W (7) x. W
$\square_i(x)$
( ) $f_i(x \square D_{-i})$.

A DGP
(1)
$u \square \delta$, $F_{\varepsilon|x}$.[11] I BNE -
, N (7)
. W
- S 4.1 .

F , $N = 2$, BNE x -
. I ,

11

i x:

$$H_i^0 : \square_i(x) = 0$$
$$H_i^1 : \square_i(x) \neq 0$$

G $i \leq N$, *f* (FWE), , . T ,

$$\mathrm{FWE}_P = P_P\{ \quad H_i^0 : \square_i(x) = 0 \quad i \in \mathbf{I}_0(P)\}$$

P DGP $\mathbf{I}_0(P) \subset \{1, \ldots, N\}$ i P. A - - FWE$_P$ α $_{G\to+\infty}$ FWE$_P \leq \alpha$ P. W Ω_X x . S x - $X = x$. W Ω_X , X - - (. .,). S , Ω_X . N

$\hat{\boldsymbol{\mu}}_G$ … μ. B … , $G^{1/2}(\hat{\boldsymbol{\mu}}_G(\{x\}) - \boldsymbol{\mu}(\{x\})) \xrightarrow{d} \mathcal{N}(\mathbf{0}_{\tilde{N}}, \boldsymbol{\Sigma}(\{x\}))$ … $G \to \infty$, … $\mathbf{0}_{\tilde{N}}$ … $\tilde{N}$ … Σ … . D … $\mathbf{T}_G(\{x\})$ … N … i …

$$T_{G,i}(\{x\}) \equiv \sum_{j \neq i} \frac{\hat{\mu}_{ij}(\{x\})}{\hat{\mu}_0(\{x\})} - \frac{\hat{\mu}_i(\{x\})\hat{\mu}_j(\{x\})}{(\hat{\mu}_0(\{x\}))^2}$$

B … , …

$$G^{1/2}\left(\mathbf{T}_G(\{x\}) - \boldsymbol{\Delta}(x)\right) \xrightarrow{d} N\left(\mathbf{0}_N, \mathbf{V}(\{x\})\boldsymbol{\Sigma}(\{x\})\mathbf{V}(\{x\})'\right)$$

… $G \to \infty$

… $\boldsymbol{\Delta}(x) \equiv (\Delta_i(x))_{i=1}^N$. T … J … $\mathbf{V}(\{x\})$ … N- … -$\tilde{N}$ … , … i … $V_i(\{x\})$ … (… $\mu_{(m)}(\{x\})$, $V_{i,(m)}(\{x\})$ … m … $\tilde{N}$ … $\boldsymbol{\mu}(\{x\})$ … $V_i(\{x\})$, … , … $j, k \neq i$):

$\mu_{(m)}(\{x\})$	$V_{i,(m)}(\{x\})$
$\mu_0(\{x\})$:	$\sum_{j\neq i}\left(-\frac{\mu_{ij}(\{x\})}{\mu_0(\{x\})^2} + \frac{2\mu_i(\{x\})\mu_j(\{x\})}{\mu_0(\{x\})^3}\right)$
$\mu_i(\{x\})$:	$-\sum_{j\neq i}\frac{\mu_j(\{x\})}{\mu_0(\{x\})^2}$
$\mu_j(\{x\})$:	$-\frac{\mu_i(\{x\})}{\mu_0(\{x\})^2}$
$\mu_{ij}(\{x\})$ … $\mu_{ji}(\{x\})$:	$\frac{1}{\mu_0(\{x\})}$
$\mu_{jk}(\{x\})$:	0

W … $\boldsymbol{\Sigma}(\{x\})$, $\mathbf{V}(\{x\})$ … $\mu_0(\{x\}), \mu_I(\{x\})$ … . F … , … $(\{x\})$ … .

W … B … H … ' … . B … p- … (… - … i) … . W … p- … $\hat{p}_{G,i}$. T … B … α … i … $\hat{p}_{G,i} \leq \alpha/N$. T … H … ' … , … - … B … , … . (F … , … x.) T … H … ' … p- … - … : $\hat{p}_{G,(1)} \leq \hat{p}_{G,(2)} \leq \cdots \leq \hat{p}_{G,(N)}$. L … $H^0_{j_k}$:

.[15] W $\hat{c}_k$ S -

S ' ,

$$\left(\hat{V}_i(Y_i(x))\hat{\Sigma}(Y_i(x))\hat{V}_i(Y_i(x))'/G\right)^{-1/2} \times \left(T_{G,i}(Y_i(x)) - \Psi_i(x)\right) \xrightarrow{d} \mathcal{N}(0,1) \quad G \to \infty$$

$\hat{V}_i(Y_i(x))$ $\hat{\Sigma}(Y_i(x))$ $V_i(Y_i(x))$ $\Sigma(Y_i(x))$, S 4.1. T $\delta_i(x)$

F ... (λ, G), ...

$$D_{i,g} = 1\left[u_i - W_g \sum_{j \neq i} p_j^1 - (1 - W_g) \sum_{j \neq i} p_j^2 - \varepsilon_{i,g} \geq 0\right]$$

... , ... $g \leq G$, W_g ... B ... λ, $\varepsilon_{i,g}$ ... $\mathcal{N}(0, 1, 0.25^2)$ ... p^l' ... B ... N ... F ... (λ, G), ... $S = 1000$ ... F ... , ... - ... - ... S 4.2 ... - ... W ... - ... $\hat{c}_k$ S 4.2: ( ) ... T_G, ( ) ... , ... ( ) ... (A 3.2 4.2 R ... W (2005)). F ... , ... - ... ( ) ... ( ) ... ( ) (... B). W ... $B = 1000, 2000$. I T ... I, ... (... ., ... H_0 ... $i = 1$) ... $S = 1000$ ... RP1, RP2, ... RP3, ... RP ... .

[redacted]	$i = 3$
[redacted]	[0.000, 1.000]
[redacted]	[0.000, 1.000]
[redacted]	[0.000,1.000]
[redacted]	[0.000, 1.000]
[redacted]	[0.000, 1.000]
[redacted]	[0.000, 1.000]

[redacted]. T. H_0 H_-.

[redacted] i -

[redacted] (

TABLE IV

FINITE SAMPLE PERFORMANCE: TEST OF SIGNS OF INTERACTION EFFECTS

	$G = 5000$			$G = 10{,}000$		
	$\delta = 0.8$	$\delta = 0.9$	$\delta = 1.0$	$\delta = 0.8$	$\delta = 0.9$	$\delta = 1.0$
$X_1 = -1$	[0.000, 0.469]	[0.001, 0.628]	[0.000, 0.854]	[0.000, 0.717]	[0.000, 0.890]	[0.000, 0.986]
$X_2 = -1/2$	[0.003, 0.359]	[0.000, 0.520]	[0.000, 0.714]	[0.000, 0.577]	[0.000, 0.790]	[0.000, 0.925]
$X_3 = -1$	[0.000, 0.483]	[0.000, 0.643]	[0.000, 0.834]	[0.000, 0.702]	[0.000, 0.888]	[0.000, 0.986]
$X_1 = 2$	[0.323, 0.004]	[0.459, 0.000]	[0.667, 0.000]	[0.484, 0.000]	[0.736, 0.000]	[0.910, 0.000]
$X_2 = 3/2$	[0.400, 0.000]	[0.617, 0.000]	[0.817, 0.000]	[0.665, 0.000]	[0.867, 0.000]	[0.979, 0.000]
$X_3 = 3$	[0.300, 0.004]	[0.496, 0.000]	[0.735, 0.000]	[0.545, 0.000]	[0.764, 0.000]	[0.930, 0.000]

N ... $S = 1000$. T ... $[q_+, q_-]$: q_+ ... H_0 ... H_+; q_- ... H_0 ... H_-.

... 1 ... x. A ..., $\Lambda^*_{x_i}(p) = 1/4$ ... $p \in \{(0.3233, 0.5603, 0.3233), (0.2523, 0.5288, 0.7098), (0.2998, 0.7013, 0.2998), (0.2101, 0.7262, 0.7231)\}$, ... T ... 1 ... , ... 1. A ... X_1

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TABLE VI

MULTIPLICITY TESTS (X = HOUR OF DAY)

		:55 . . . . :55	G
A	W	33.32*	26,152
	RW	$T_1^\dagger > T_3^\dagger > T_2^\dagger > 0$	
N 1	W	3.86	6534
	RW	$T_3 > T$	

TABLE VII

:55 MIN VS. NOT :55 MIN (X = HOUR OF DAY, MARKET SIZE)

M S ( )		H D			
		N 1	4 5	5 6	9 10
1	W	0.77	4.94	3.22	2.27
	RW	$T_3 > T_2 > 0 > T_1$	$T_2 > T_1 > T_3 > 0$	$T_2 > T_1 > T_3 > 0$	$T_1 > T_3 > T_2 > 0$
	G	2201	2201	2200	2199
2	W	0.73	3.87	1.97	2.48
	RW	$T_2 > T_3 > 0 > T_1$	$T_3 > 0 > T_1 > T_2$	$T_2 > T_1 > T_3 > 0$	$T_2 > T_1 > T_3 > 0$
	G	2157	2220	2159	2153
3	W	4.96	19.06*	26.07*	2.92
	RW	$T_2 > T_3 > T_1 > 0$	$T_3^\dagger > T_2^\dagger > T_1^\dagger > 0$	$T_1^\dagger > T_3^\dagger > T_2^\dagger > 0$	$T_1 > T_3 > 0 > T_2$
	G	2176	2141	2177	2168

T (∗) W 5%. T (†) - 5% FWE R W (2005) (RW). T_k ( ) k. T 1000.

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7. CONCLUSION

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